Reducing Ringing Artefact in Fresnel Digital Holography Using Compressed Sensing

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ABSTRACT

Compressed sensing is a signal processing technique used for signal reconstruction with significantly smaller number of samples than the requirements of the Nyquist-Shannon theorem. In this work, we simulate a lenseless digital holographic system. We investigate the ringing-like artefact introduced by truncation by the camera aperture. We present the results of using the orthogonal matching pursuit based compressed sensing algorithms to combat this ringing-like artefact. We demonstrate that compressed sensing achieves remarkable reconstructions and suppresses ringing well, but only up to a point in terms of the size of the aperture. This research could help the advancement of compressive digital holography.

Keywords: Gibbs ringing, Compressive sensing, Fresnel Digital holography

1. INTRODUCTION

Compressed sensing (CS) is a signal processing technique that has the ability to reconstruct a signal with fewer samples than the requirement of the Shannon-Nyquist theorem. It was first proposed by Emmanuel Candès, Justin Romberg, Terence Tao, and David Donoho.[1–3] Before the invention of CS methods, image compression algorithms allowed most of the acquired data to be eventually discarded with close to no perceptual loss. This phenomenon leads the creators of CS to ask the question of whether it is possible to not measure the eventually discarded information in the first place.[1] CS achieve this by using optimization methods to exploit the sparsity of a signal. There are two main conditions that have to be met for CS to recover the signal.[4] The first is that we need to be able to assume that the signal has a sparse representation in some domain. We can use a sparsifying operator to transform the signal to that domain. The second is that the object need to be projected to another signal space. The projection transform needs to have low similarity with the sparsifying operator.

Several categories of algorithms can be used to perform the CS reconstruction.[5] The convex optimization methods use linear programming solvers such as basis pursuit to obtain the sparse representation of a signal.[6] Greedy approaches such as Orthogonal Matching Pursuit (OMP) use iterations to find the most correlated columns in the mixing matrix to the measurements.[7] The iterative hard/soft thresholding algorithm use a thresholding operation to simultaneously operate on k columns in the mixing matrix.[8,9] Recently, machine learning methods have also been proposed for finding the CS reconstruction.[10,11]

CS has been adopted by a various areas of study. One area where CS could be applied is digital holography (DH).[4] After Brady *et al.*[12] successfully introduced CS in digital holography, CS was applied under various digital holographic systems. A holographic setup has the ability to capture a 3D digital hologram of an object. Reconstruction algorithms then reconstruct a 3D image using the recorded phase and amplitude information of the object. One type of digital holography is Fresnel digital holography. This type of setup uses the Fresnel transform. The Fresnel transform is a special case of the linear canonical transform[13] and is a model for free space propagation. Compressive sensing was first also applied to digital Fresnel holography by Rivenson *et al.*[14]

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Because we would like to test the CS algorithm on a Fresnel digital holography system, the sensing matrix in our CS algorithm is a Fresnel transform matrix. The combination of the Fresnel transform and the Haar wavelet transform have low coherence,[4] which is why we used the Haar wavelet transform to sparsify the obejct.

When a discontinuous signal is approximated by its Fourier transform, ringing artefacts occur at the locations of discontinuities. This ringing artefact is known as Gibbs ringing. We have previously investigated the effect of Gibbs ringing and its suppression.[15,16] Commonly used methods for Gibbs suppression include low-pass filtering, subvoxel shift,[17] Gegenbauer reconstruction[18] and machine learning methods.[19,20] In this work, we investigate how CS could be used to suppress a ringing-like artefact in Fresnel transform, shown in.[21]

The structure of this paper is as follows. We summarize the theory behind orthogonal matching pursuit (OMP) in Sec. 2. The analysis of compressed sensing and Gibbs ringing is shown in Sec. 3. Finally, we present our conclusions in Sec. 4.

2. THEORY

We can summarize the compressive sensing technique into Eq. 1.

$$y = \phi f^* = \phi\psi x = Ax \tag{1}$$

where y is the vectorized truncated Fresnel data, ϕ is the Fresnel sensing matrix, ψ is the sparsifying matrix, f^* is the vectorized original test hologram, A is the mixing matrix and x is what we try to find using the OMP algorithm. Assuming that the signal we wish to reconstruct is sparse in the wavelet domain, then only a few linear projections of the signal could be used to reconstruct the hologram. A more detailed explanation of compressed sensing theory could be found in.[5,22]

In a minimum L_1 norm solution using linear programming(LP), we seek:

$$\hat{x} \triangleq \arg\min_{x}(||x||_1, \text{ subject to: } y = Ax) \tag{2}$$

The algorithm that we used to solve the optimization problem was orthogonal matching pursuit (OMP). It is an iterative greedy method that could be used to find the sparsest representation. It was first proposed by Pati *et al.* in 1993.[7] As shown in Eq. 2, the core problem in CS is to solve for x in $y = Ax$ with prior knowledge of y and A. To solve this, OMP assumes x to be sparse and treat y as a sparse linear combination of columns of A.

At the beginning of OMP, two initialization steps are required. We need to set A_0, the index set, to be zeros the same size as A, and residual $r = y$. After the initialization is done, the OMP algorithm will iterate k times. In each iteration of OMP, the index set A_0 is first updated by selecting a column in A that are the most correlated to the residual r. Then, that column in A is extracted and the same column will not be extracted again in later iterations. We then calculate x using $x = A^{\dagger}y$ where $A^{\dagger}$ is the Moore–Penrose psudoinverse of A_0. Finally, the residual r is updated by $r = y - \hat{A}x$ where $\hat{A}$ is the mixing matrix A without all the selected columns. After the residual r is updated, another iteration will begin.

3. COMPRESSED SENSING AND GIBBS RINGING

In this section, we use a rectangular test hologram to demonstrate the usage of compressive sensing in Fresnel digital holography and their ability to suppress ringing. We present simulations of Fresnel digital holographic reconstructions using the setup depicted in.[21] In this simulated system, we use the Fresnel transform as our sensing matrix ϕ. In the Fresnel transform matrix, the wavelength of the light beam is $\lambda = 632.8$ nm to simulate a red helium-neon laser light in air. The propagation distance is 0.3 m and the sampling distance between pixels is $T = 10$ micron. The resulting hologram has 32×32 pixels. A Haar wavelet transform was used as the sparsifying matrix, ψ^{-1}.

In Fig. 1, we show that CS has the ability to recover information using limited Fresnel data simulating the apodization process. In this figure, the top row shows the absolute value of three holograms, showing their amplitude. The row in the middle row shows the top row's corresponding spectrum. Within these two rows, (a)

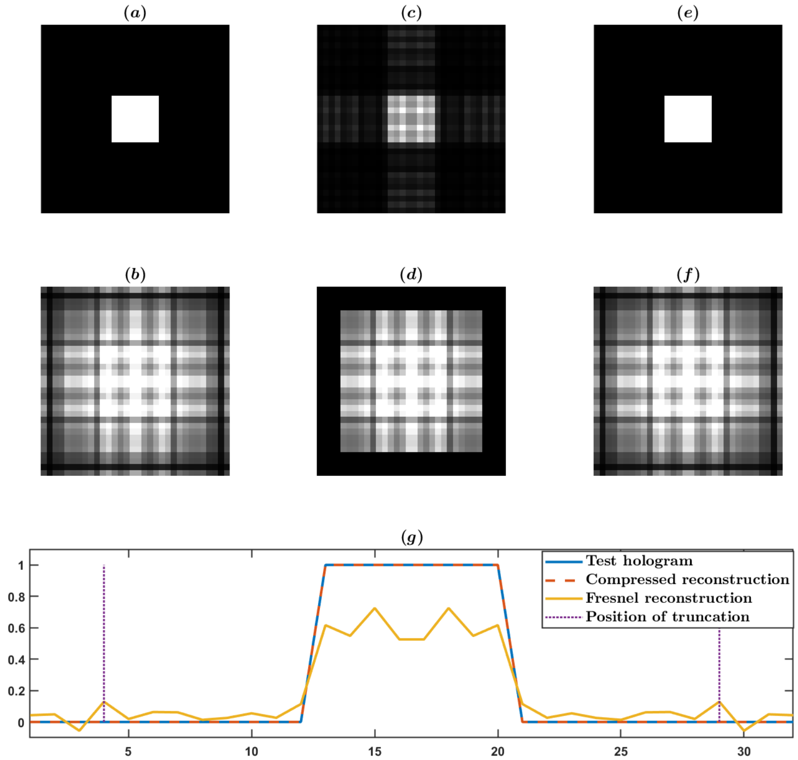

Figure 1. In this figure, (a) shows the amplitude of the original test hologram. (b) is the spectrum of (a). (c) is the reconstructed hologram using the truncated and zeropadded spectrum shown in (d). (e) is the OMP-based compressive reconstruction using (d) as input, and (f) is its spectrum's absolute value. (g) shows a slice through the middle of (a),(c) and (e) with purple dotted line indicating the position of the truncation.

and (b) shows the test hologram and its spectrum. (d) shows the spectrum after apodization that removes the outermost 4 pixels on all sides and zeropadding that fills 0s in these positions. We refer to this measurement of 4 pixels as the *truncation*. (c) is the Fresnel reconstruction of (d). (e) is the CS reconstructed image and we plotted its spectrum in (f) for reference. The bottom plot (g) shows a slice through the middle of the three holograms in the top row, showing the test hologram (blue) ringing-like artefact in the Fresnel reconstruction (yellow) and the results of CS reconstruction (red dashed). The positions of apodization are shown in purple dotted line in (g). Unlike Gibbs ringing in Fourier reconstruction that only appears around discontinuities, it is worth pointing out that the ringing-like artefact in Fresnel transform also appears at the locations of apodization(purple dotted). It can be seen that the CS reconstruction completely eliminates the ringing in this example, which demonstrates that CS has the ability to restore the hologram with limited Fresnel domain data.

Even though CS has been shown to have the ability to combat the aftermath of apodization in Fig 1, the superiority of CS is limited to how much truncation that was introduced in the Fresnel domain. In Fig. 2, we plot the structural similarity index measure (SSIM) of both CS and Fresnel reconstruction as a function of truncation to test this limit of our CS algorithm. It can be seen that CS reaches its limit after truncation gets larger. It is only capable of perfect reconstruction up to a point, after which it starts to degrade.

In Fig. 2, the SSIM between the CS reconstruction and the test hologram drops below 1 when truncation is greater than 8. In Fig. 3, we present a simulation where the CS reconstruction is just starting to fail with truncation equals to 9.

4. CONCLUSIONS

We simulated a Fresnel digital holographic system in this work. The system has a limiting camera aperture that introduces ringing in the hologram. We reconstructed the hologram using zero-padded Fresnel reconstruction and

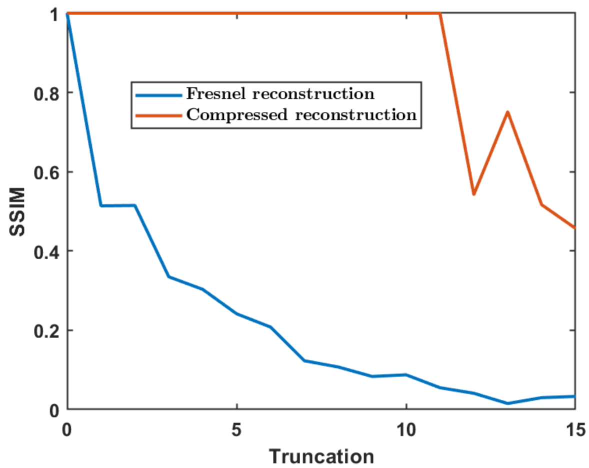

Figure 2. In this figure, SSIM of the reconstruction for both Fresnel reconstruction and compressed reconstruction are plotted as a function of truncation. It can be seen that the compressed reconstruction is perfect for truncation ≤ 8 and that the compressed reconstruction always has higher SSIM scores than the Fresnel reconstruction.

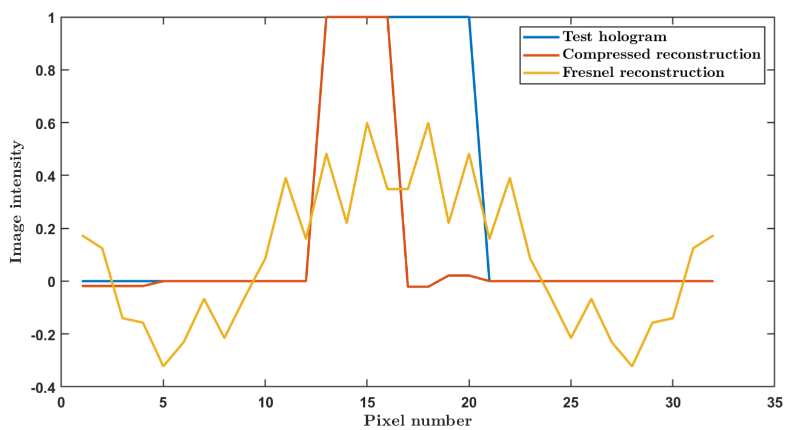

Figure 3. The blue line is a slice through the original test hologram. The red line is the same slice through the CS reconstruction and the yellow line is the same slice through the Fourier reconstruction. This shows a case in which the CS algorithm is just starting to fail.

an OMP based CS reconstruction. We investigated the possibility of using CS to suppress ringing-like artefact in Fresnel digital holography. We believe that this is the first exploration of using CS as a ringing suppression method for Fresnel digital holography.

We have discovered that CS has the ability to suppress the ringing-like artefact in Fresnel digital holography, but up to a limit in terms of truncation. When the limit of truncation is passed, CS reconstruction still has a superior SSIM score, but degrades as much as Fresnel reconstruction does. We believe that using CS methods to increase resolution first and then use other zoom algorithms and Gibbs suppression methods to further increase the resolution would be the best approach in that situation.

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REFERENCES

[1] Donoho, D. L., "Compressed sensing," *IEEE Transactions on information theory* **52**(4), 1289–1306 (2006).

[2] Candes, E. J., Romberg, J. K., and Tao, T., "Stable signal recovery from incomplete and inaccurate measurements," *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences* **59**(8), 1207–1223 (2006).

[3] Candès, E. J. and Wakin, M. B., "An introduction to compressive sampling," *IEEE signal processing magazine* **25**(2), 21–30 (2008).

[4] Rivenson, Y., Stern, A., and Javidi, B., "Overview of compressive sensing techniques applied in holography," *Applied optics* **52**(1), A423–A432 (2013).

[5] Rani, M., Dhok, S. B., and Deshmukh, R. B., "A systematic review of compressive sensing: Concepts, implementations and applications," *IEEE access* **6**, 4875–4894 (2018).

[6] Chen, S. S., Donoho, D. L., and Saunders, M. A., "Atomic decomposition by basis pursuit," *SIAM review* **43**(1), 129–159 (2001).

[7] Pati, Y. C., Rezaiifar, R., and Krishnaprasad, P. S., "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in [*Proceedings of 27th Asilomar conference on signals, systems and computers*], 40–44, IEEE (1993).

[8] Blumensath, T. and Davies, M. E., "Iterative hard thresholding for compressed sensing," *Applied and computational harmonic analysis* **27**(3), 265–274 (2009).

[9] Daubechies, I., Defrise, M., and De Mol, C., "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences* **57**(11), 1413–1457 (2004).

[10] Wu, Y., Rosca, M., and Lillicrap, T., "Deep compressed sensing," in [*International Conference on Machine Learning*], 6850–6860, PMLR (2019).

[11] Shi, W., Jiang, F., Liu, S., and Zhao, D., "Image compressed sensing using convolutional neural network," *IEEE Transactions on Image Processing* **29**, 375–388 (2019).

[12] Brady, D. J., Choi, K., Marks, D. L., Horisaki, R., and Lim, S., "Compressive holography," *Optics express* **17**(15), 13040–13049 (2009).

[13] Healy, J. J., Kutay, M. A., Ozaktas, H. M., and Sheridan, J. T., [*Linear canonical transforms: Theory and applications*], vol. 198, Springer (2015).

[14] Rivenson, Y., Stern, A., and Javidi, B., "Compressive Fresnel holography," *Journal of Display Technology* **6**(10), 506–509 (2010).

[15] Wang, Y. and Healy, J. J., "Automated filter selection for suppression of Gibbs ringing artefacts in MRI," *Magnetic Resonance Imaging* **93**, 3–10 (2022).

[16] Wang, Y. and Healy, J. J., "Image quality assessment for Gibbs ringing reduction," *Algorithms* **16**(2), 96 (2023).

[17] Kellner, E., Dhital, B., Kiselev, V. G., and Reisert, M., "Gibbs-ringing artifact removal based on local subvoxel-shifts," *Magnetic resonance in medicine* **76**(5), 1574–1581 (2016).

[18] Gottlieb, D., Shu, C.-W., Solomonoff, A., and Vandeven, H., "On the Gibbs phenomenon I: recovering exponential accuracy from the Fourier partial sum of a nonperiodic analytic function," *Journal of Computational and Applied Mathematics* **43**(1-2), 81–98 (1992).

[19] Muckley, M. J., Ades-Aron, B., Papaioannou, A., Lemberskiy, G., Solomon, E., Lui, Y. W., Sodickson, D. K., Fieremans, E., Novikov, D. S., and Knoll, F., "Training a neural network for Gibbs and noise removal in diffusion MRI," *Magnetic resonance in medicine* **85**(1), 413–428 (2021).

[20] Zhang, Q., Ruan, G., Yang, W., Liu, Y., Zhao, K., Feng, Q., Chen, W., Wu, E. X., and Feng, Y., "MRI Gibbs-ringing artifact reduction by means of machine learning using convolutional neural networks," *Magnetic resonance in medicine* **82**(6), 2133–2145 (2019).

[21] Wang, Y. and Healy, J. J., "Gibbs ringing and the Fresnel transform," in [*Optics, Photonics and Digital Technologies for Imaging Applications VII*], **12138**, 121380T, SPIE (2022).

[22] Wang, Y. and Healy, J. J., "Compressive digital holography and Gibbs ringing," in [*Holography: Advances and Modern Trends VIII*], **12574**, 12574–31, SPIE (2023).